EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 5.6

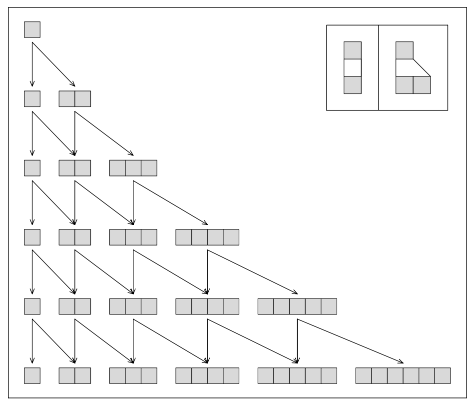
Multiway Systems

Multiway Systems

The network systems that we discussed in the previous section do not have any underlying grid of elements in space. But they still in a sense have a simple one-dimensional arrangement of states in time. And in fact, all the systems that we have considered so far in this book can be thought of as having the same simple structure in time. For all of them are ultimately set up just to evolve progressively from one state to the next.

Multiway systems, however, are defined so that they can have not just a single state, but a whole collection of possible states at any given step.

The picture below shows a very simple example of such a system.

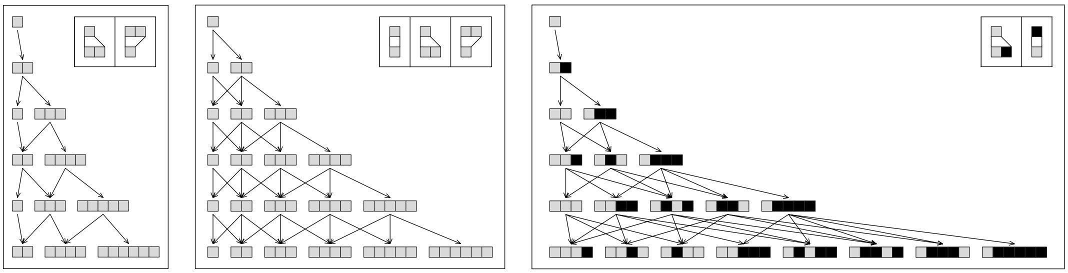


A very simple multiway system in which one element in each sequence is replaced at each step by either one or two elements. The main feature of multiway systems is that all the distinct sequences that result are kept.

Each state in the system consists of a sequence of elements, and in the particular case of the picture above, the rule specifies that at each step each of these elements either remains the same or is replaced by a pair of elements. Starting with a single state consisting of one element, the picture then shows that applying these rules immediately gives two possible states: one with a single element, and the other with two.

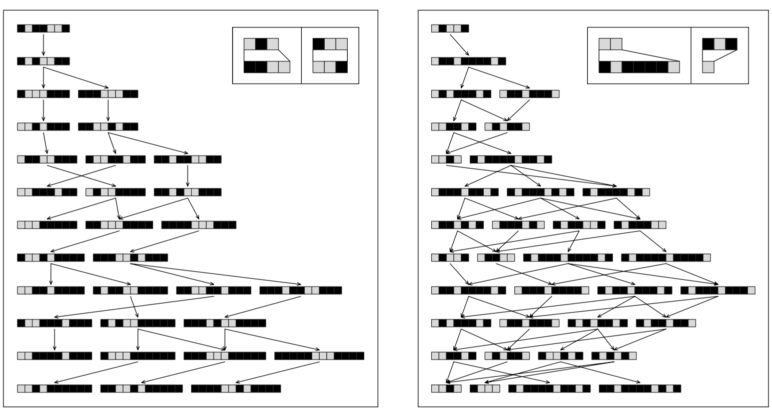
Multiway systems can in general use any sets of rules that define replacements for blocks of elements in sequences. We already saw exactly these kinds of rules when we discussed sequential substitution systems on page 88. But in sequential substitution systems the idea was to do just one replacement at each step. In multiway systems, however, the idea is to do all possible replacements at each step—and then to keep all the possible different sequences that are generated.

The pictures below show what happens with some very simple rules. In each of these examples the behavior turns out to be rather simple—with for example the number of possible sequences always increasing uniformly from one step to the next.

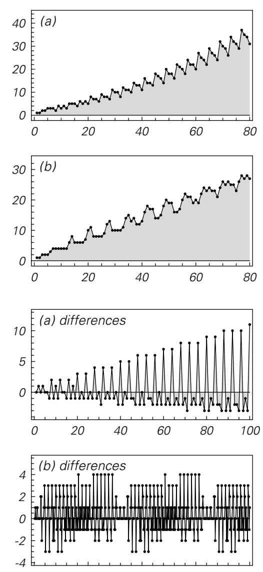


Examples of simple multiway systems. The number of distinct sequences at step t in these three systems is respectively *Ceiling[t/2]*, t and *Fibonacci[t + 1]* (which increases approximately like 1.618^t).

In general, however, this number need not exhibit such uniform growth, and the pictures below show examples where fluctuations occur.

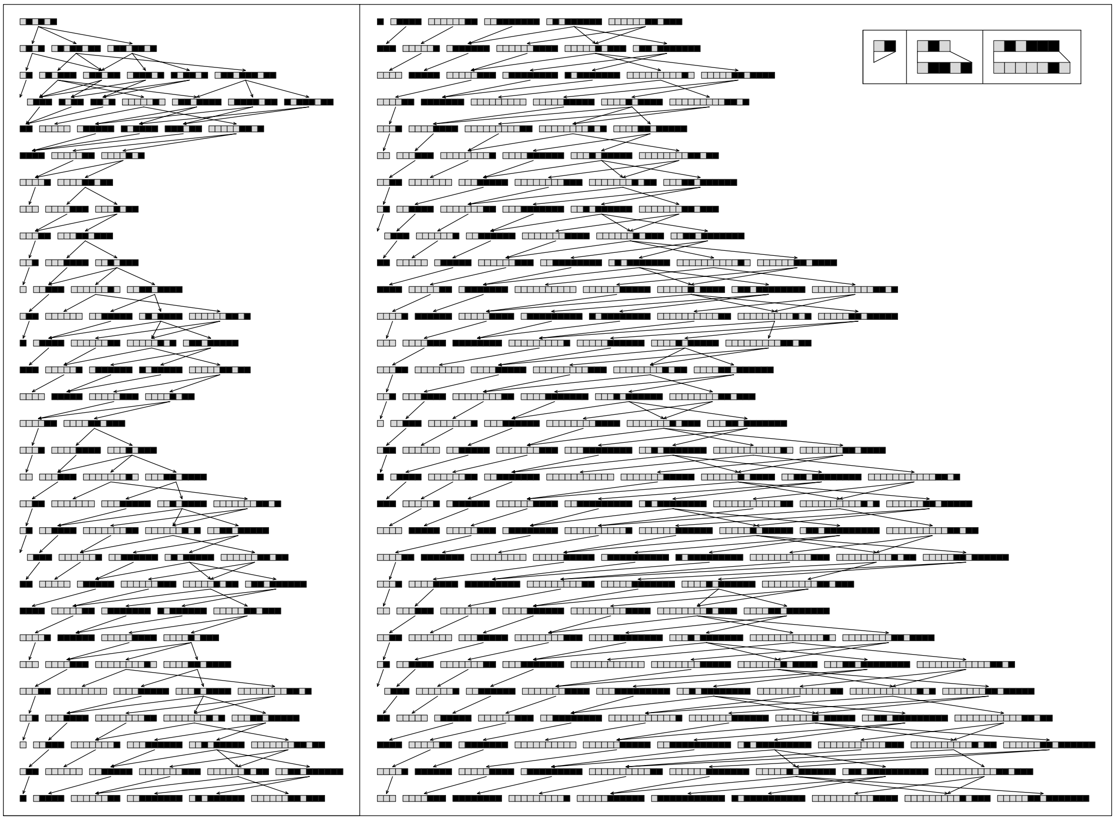


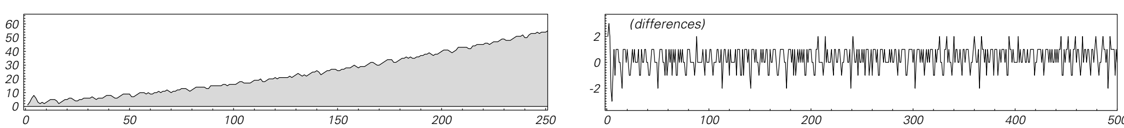
(a) (b)



Examples of multiway systems with slightly more complicated behavior. The plots on the right show the total number of possible states obtained at each step, and the differences of these numbers from one step to the next. In both cases, essentially repetitive behavior is seen, every 40 and 161 steps respectively. Note that in case (a), the total number of possible states at step t increases roughly like t^2, while in case (b) it increases only like t.

But in both these cases it turns out to be not too long before these fluctuations essentially repeat. The picture below shows an example where a larger amount of apparent randomness is seen. Yet even in this case one finds that there ends up again being essential repetition—although now only every 1071 steps.

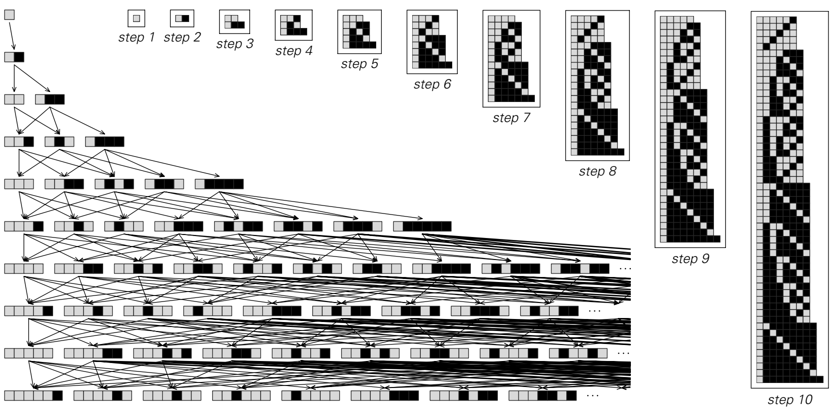


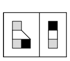


A multiway system with behavior that shows some signs of apparent randomness. The rule for this system involves three possible replacements. Note that the first replacement only removes elements and does not insert new ones. In the pictures sequences containing zero elements therefore sometimes appear. At least with the initial condition used here, despite considerable early apparent randomness, the differences in number of elements do repeat (shifted by 1) every 1071 steps.

If one looks at many multiway systems, most either grow exponentially quickly, or not at all; slow growth of the kind seen on the facing page is rather rare. And indeed even when such growth leads to a certain amount of apparent randomness it typically in the end seems to exhibit some form of repetition. If one allows more rapid growth, however, then there presumably start to be all sorts of multiway systems that never show any such regularity. But in practice it tends to be rather difficult to study these kinds of multiway systems—since the number of states they generate quickly becomes too large to handle.

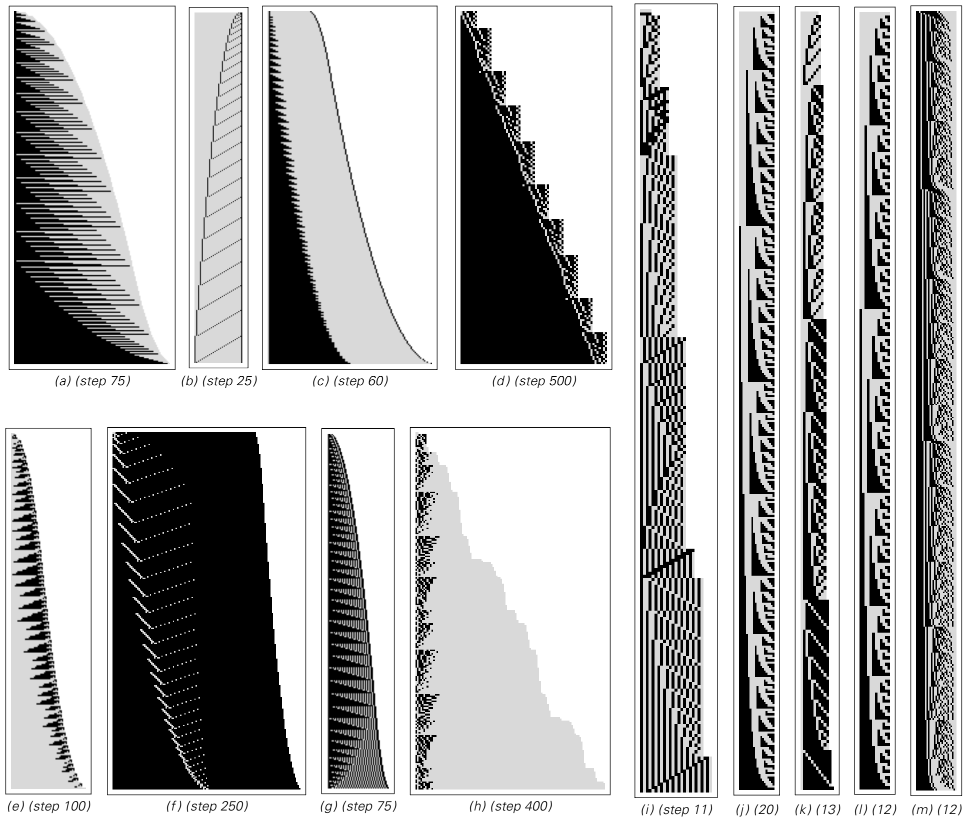
One can get some idea about how such systems behave, however, just by looking at the states that occur at early steps. The picture below shows an example—with ultimately fairly simple nested behavior.



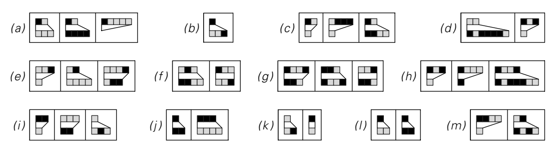


The collections of states generated on successive steps by a simple multiway system with rapid growth shown on page 205. The particular rule used here eventually generates all states beginning with a white cell. At step *t* there are *Fibonacci[t + 1]* states; a given state with *m* white cells and *n* black cells appears at step $2m + n - 1$.

The pictures on the next page show some more examples. Sometimes the set of states that get generated at a particular step show essential repetition—though often with a long period. Sometimes this set in effect includes a large fraction of the possible digit sequences of a given length—and so essentially shows nesting. But in other cases there is at least a hint of considerably more complexity—even though the total number of states may still end up growing quite smoothly.

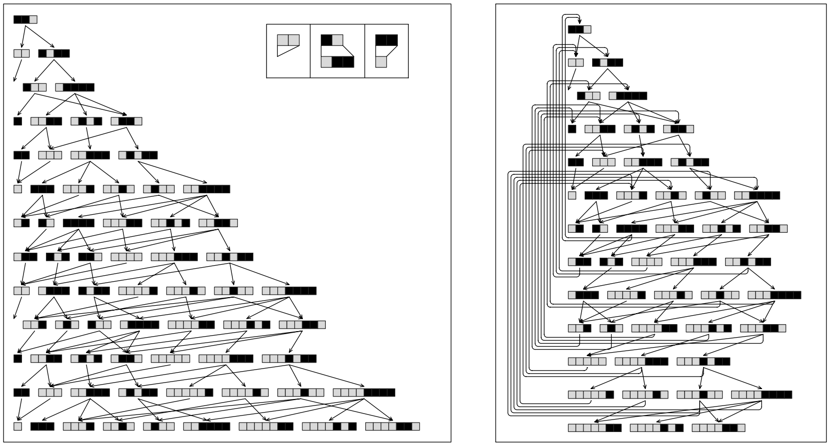


Collections of states generated at particular steps in the evolution of various multiway systems. Rule (k) was shown on the previous page; rules (d) and (f) on page 205.



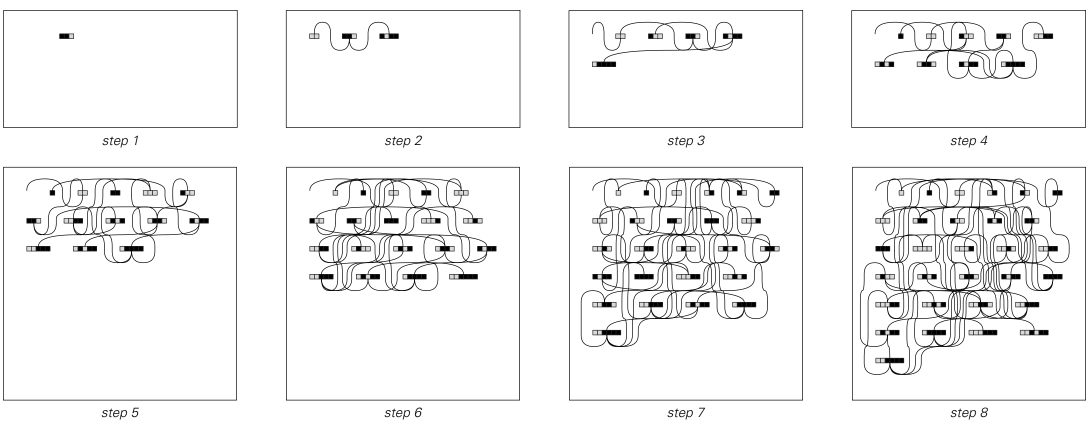
Looking carefully at the pictures of multiway system evolution on previous pages, a feature one notices is that the same sequences often occur on several different steps. Yet it is a consequence of the basic setup for multiway systems that whenever any particular sequence occurs, it must always lead to exactly the same behavior.

So this means that the complete evolution can be represented as in the picture at the top of the facing page, with each sequence shown explicitly only once, and any sequence generated more than once indicated just by an arrow going back to its first occurrence.



The evolution of a multiway system, first with every sequence explicitly shown at each step, and then with every sequence only ever shown once.

But there is no need to arrange the picture like this: for the whole behavior of the multiway system can in a sense be captured just by giving the network of what sequence leads to what other. The picture below shows stages in building up such a network. And what we see is that just as the network systems that we discussed in the previous section can build up their own pattern of connections in space, so also multiway systems can in effect build up their own pattern of connections in time—and this pattern can often be quite complicated.



The network built up by the evolution of the multiway system from the top of the page. This network in effect represents a network of connections in time between states of the multiway system.