EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 11.10

Class 4 Behavior and Universality

The practical importance of this phenomenon depends greatly however on how far one has to go to get to the threshold of universality.

But knowing that a system like rule 110 is universal, one now suspects that this threshold is remarkably easy to reach. And what this means is that beyond the very simplest rules of any particular kind, the behavior that one sees should quickly become as complex as it will ever be.

Remarkably enough, it turns out that this is essentially what we already observed in Chapter 3. Indeed, not only for cellular automata but also for essentially all of the other kinds of systems that we studied, we found that highly complex behavior could be obtained even with rather simple rules, and that adding further complication to these rules did not in most cases noticeably affect the level of complexity that was produced.

So in retrospect the results of Chapter 3 should already have suggested that simple underlying rules such as rule 110 might be able to achieve universality. But what the elaborate construction in the previous section has done is to show for certain that this is the case.

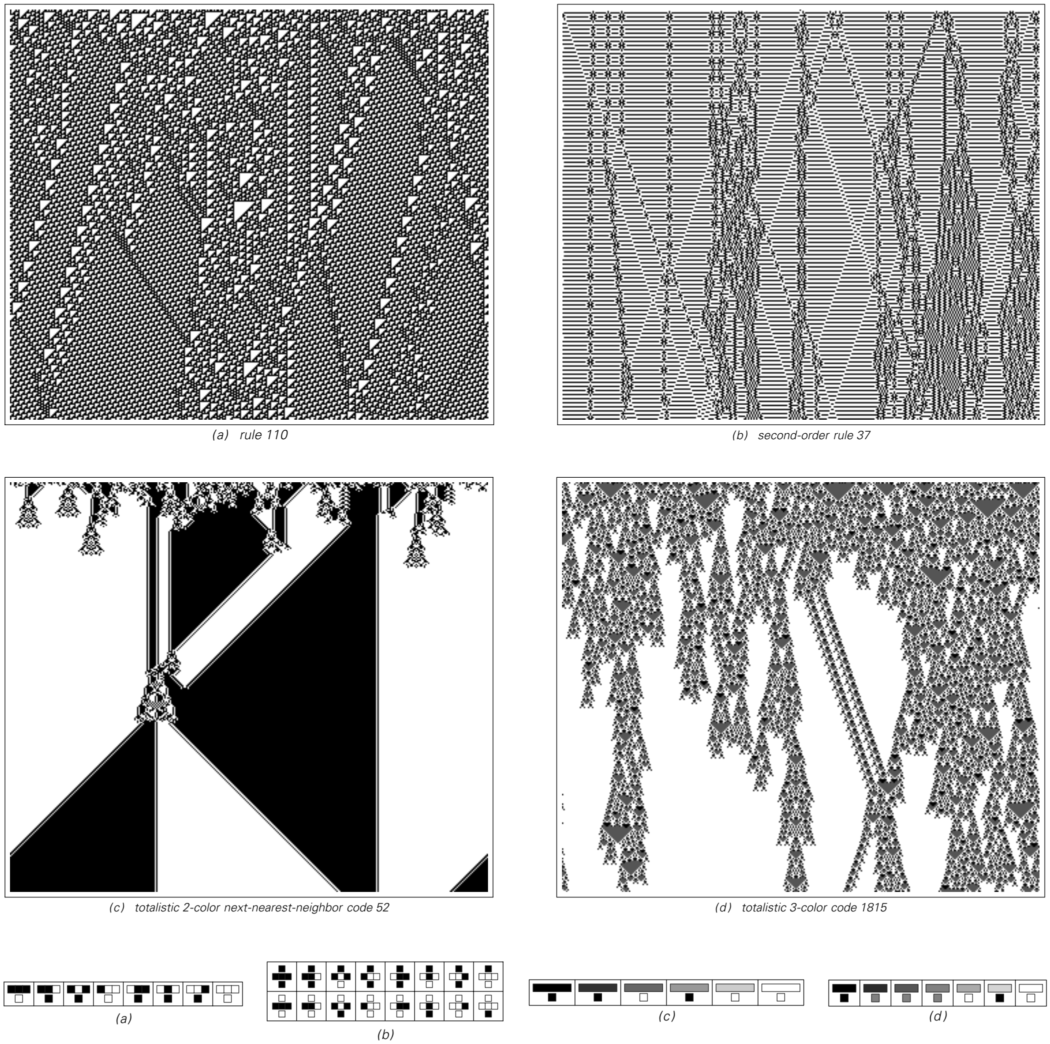
Class 4 Behavior and Universality

If one looks at the typical behavior of rule 110 with random initial conditions, then the most obvious feature of what one sees is that there are a large number of localized structures that move around and interact with each other in complicated ways. But as we saw in Chapter 6, such behavior is by no means unique to rule 110. Indeed, it is in fact characteristic of all cellular automata that lie in what I called class 4.

The pictures on the next page show a few examples of such class 4 systems. And while the details are different in each case, the general features of the behavior are always rather similar.

So what does this mean about the computational capabilities of such systems? I strongly suspect that it is true in general that any cellular automaton which shows overall class 4 behavior will turn out—like rule 110—to be universal.

We saw at the end of Chapter 6 that class 4 rules always seem to yield a range of progressively more complicated localized structures. And my expectation is that if one looks sufficiently hard at any



Examples of cellular automata with class 4 overall behavior, as discussed in Chapter 6. I strongly suspect that all class 4 rules, like rule 110, will turn out to be universal.

particular rule, then one will always eventually be able to find a set of localized structures that is rich enough to support universality.

The final demonstration that a given rule is universal will no doubt involve the same kind of elaborate construction as for rule 110.

But the point is that all the evidence I have so far suggests that for any class 4 rule such a construction will eventually turn out to be possible.

So what kinds of rules show class 4 behavior?

Among the 256 so-called elementary cellular automata that allow only two possible colors for each cell and depend only on nearest neighbors, the only clear immediate example is rule 110—together with rules 124, 137 and 193 obtained by trivially reversing left and right or black and white. But as soon as one allows more than two possible colors, or allows dependence on more than just nearest neighbors, one immediately finds all sorts of further examples of class 4 behavior.

In fact, as illustrated in the pictures on the facing page, it is sufficient in such cases just to use so-called totalistic rules in which the new color of a cell depends only on the average color of cells in its neighborhood, and not on their individual colors.

In two dimensions class 4 behavior can occur with rules that involve only two colors and only nearest neighbors—as shown on page 249. And indeed one example of such a rule is the so-called Game of Life that has been popular in recreational computing since the 1970s.

The strategy for demonstrating universality in a two-dimensional cellular automaton is in general very much the same as in one dimension. But in practice the comparative ease with which streams of localized structures can be made to cross in two dimensions can reduce some of the technical difficulties involved. And as it turns out there was already an outline of a proof given even in the 1970s that the Game of Life two-dimensional cellular automaton is universal.

Returning to one dimension, one can ask whether among the 256 elementary cellular automata there are any apart from rule 110 that show even signs of class 4 behavior. As we will see in the next section, one possibility is rule 54. And if this rule is in fact class 4 then it is my expectation that by looking at interactions between the localized structures it supports it will in the end—with enough effort—be possible to show that it too exhibits the phenomenon of universality.